THE ROLE OF METROLOGY IN THE CASE OF THE PRECISION LEVELLING NECESSARY FOR THE REALISATION OF GRAVIMETRIC NETWORKS

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Abstract. Levelling is the method by which height differences can be determined with very high accuracy. Using these differences, it is possible to determine altitudes for certain points of interest. The gravity of a point is revealed by using gravimeters, but the calculations take into account the absolute altitude of that specific point. The higher the accuracy of the altitude determination, the greater the accuracy of the gravity measurement. For this reason, it is essential to identify the errors that accompany levelling measurements, determine which are the most significant, and establish how they can be avoided.

Key words: levelling, gravimetry, errors, sighting axis

1. PRECISION GEOMETRIC LEVELLING

1.1. Geometric levelling

Geometric levelling is the most accurate method of determining height differences. At present, over long distances of hundreds of km, the levelling achieved with the help of GNSS technology is more precise, provided that the geoid undulation at the end points is known. Geometric levelling can be of two types: from the centre and from the end. The height difference between two points is given by the difference between the readings on the back and fore staffs in the case of levelling from the centre, and by the difference between the reading on the staff and the height of the instrument in the case of levelling from the end.

Geometric levelling is widely recognised as the most accurate method for determining height differences, particularly for first- and second-order vertical control networks (Torge and Müller, 2012; Wolf and Ghilani, 2018). The method is based on horizontal sighting and precise readings on graduated staffs, allowing millimetric accuracy to be achieved under controlled conditions.

In recent years, GNSS techniques have been increasingly applied to height determination over long distances. However, the accuracy of GNSS-derived orthometric heights depends strongly on the quality of the quasigeoid or geoid models used for height transformation (Featherstone *et al.,* 1998; Hofmann-Wellenhof *et al.,* 2008). In gravimetric networks, where the vertical component must be known with high precision, geometric levelling remains indispensable. In gravimetric reductions, the Free-Air Correction is used to account for the station's height above a reference datum. Because gravity decreases as you move away from the Earth's centre, knowing the exact elevation is paramount. The importance of high-precision levelling is demonstrated by the Free-Air Correction formula, which states that gravity decreases by about 0.3086 mGal per m of height; thus, a tiny vertical error of 0.01 m translates to roughly 3.1 µGal.

The practical importance of high-precision levelling is evident not only in geodesy but also in mineral exploration or civil engineering (*e.g.* detecting voids), where a precision of 10-20 µGal is often required. Without sub-centimetre elevation data, the gravimetric survey becomes uninterpretable because the elevation noise exceeds the geological signal. Geometric levelling can be performed as levelling from the centre or levelling from the end. In the case of levelling from the centre, the instrument is positioned approximately midway between the back and fore staffs, and the height difference is obtained as the difference between the corresponding readings. This configuration minimises the influence of several systematic errors and is therefore recommended for precision levelling works (Vaníček and Krakiwsky, 1986).

1.2. Inherent errors when achieving geometric levelling

Certain errors are inherent when measuring with a level, even when using modern electronic levels and barcoded staffs. While some errors have a greater influence than others, it is essential that these are understood so that appropriate action can be taken to minimise them as much as possible.

The errors that accompany the process of determining height differences by the geometric levelling method are:

- the error due to the influence of atmospheric refraction and the curvature of the Earth;
- the residual collimation error (the non-parallelism between the sighting axis and the axis of the instrument's circular level);
- the error caused by steep gradients during levelling;
- the error caused by potential settlement of the instrument during observations;
- the error caused by potential settlement of the staff while the instrument is being moved to the next station;
- the error due to the non-verticality of the staff;
- the zero-point error of the staff.

The first three errors are inherent and cannot be entirely avoided. The remaining errors can be reduced by simple practical measures. For instance, the staff should be kept vertical during measurement, the zero-point error can be determined through laboratory calibration, and settlement can be avoided through rigorous training and procedure (Duţă, 2008; Featherstone *et al.,* 1998; INM 2020a and 2020b).

2. RESIDUAL COLLIMATION ERROR (THE NON-PARALLELISM BETWEEN THE SIGHTING AXIS AND THE AXIS OF THE INSTRUMENT'S CIRCULAR LEVEL)

Even when modern electronic levels and barcoded staffs are used, several errors accompany the determination of height differences by geometric levelling. These errors may be instrumental, observational or environmental and can affect final accuracy if not properly controlled (Ghilani, 2017). The primary errors encountered in geometric levelling were listed in Section 1.2. The first group of errors is inherent and cannot be completely eliminated, whereas the remaining errors can be reduced through proper field procedures, operator training and periodic instrument verification (Rueger, 1996; Wolf and Ghilani, 2018).
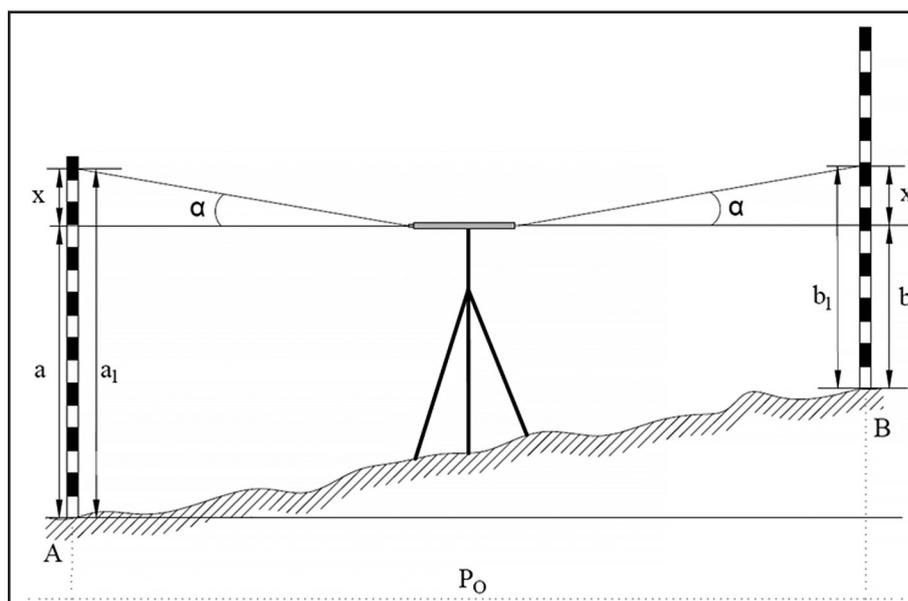

Fig. 1. Illustration of the reading error on the staff in the case of levelling from the centre (Păunescu *et al.*, 2019).

2.1. Nature of the error

The most critical error is the residual collimation error, defined as the non-parallelism between the sighting axis and the axis of the instrument's circular level. The nature of this error is illustrated in figures 1 and 2. It can be seen from figure 2 that the instrument is placed midway between the staffs. However, because the sighting axis and the axis of the circular level are not parallel, an angle α is formed between them. If the two axes were parallel, the readings on the back and fore staffs would be a and b. In this instance, as they are not parallel, the actual readings are a_1 and b_1. The height difference is calculated using formula (1), as follows:

$$\Delta h_{AB} = a_1 - b_1 \tag{1}$$

But, as can be seen from figure 2, the readings on the back and fore staffs are increased by the same quantity, denoted by x. Thus:

$$a_1 = a + x \tag{2}$$
$$b_1 = b + x$$

Substituting in formula (1) the formulas (2), we have:

$$\Delta h_{AB} = a_1 - b_1 = (a + x) - (b + x) = a - b \tag{3}$$

It follows that, by using levelling from the centre, this error is eliminated.

From figure 2, which presents the situation where the instrument is not placed perfectly at the centre, the quantities x_1 and x_2, by which the readings on the back and fore staffs are incorrect, are no longer equal. In this case, the height difference obtained is not the true one, i.e., but:

$$\Delta h_{AB} = a_1 - b_1 = (a + x_1) - (b + x_2) = (a - b) + (x_1 - x_2) \neq a - b \tag{4}$$

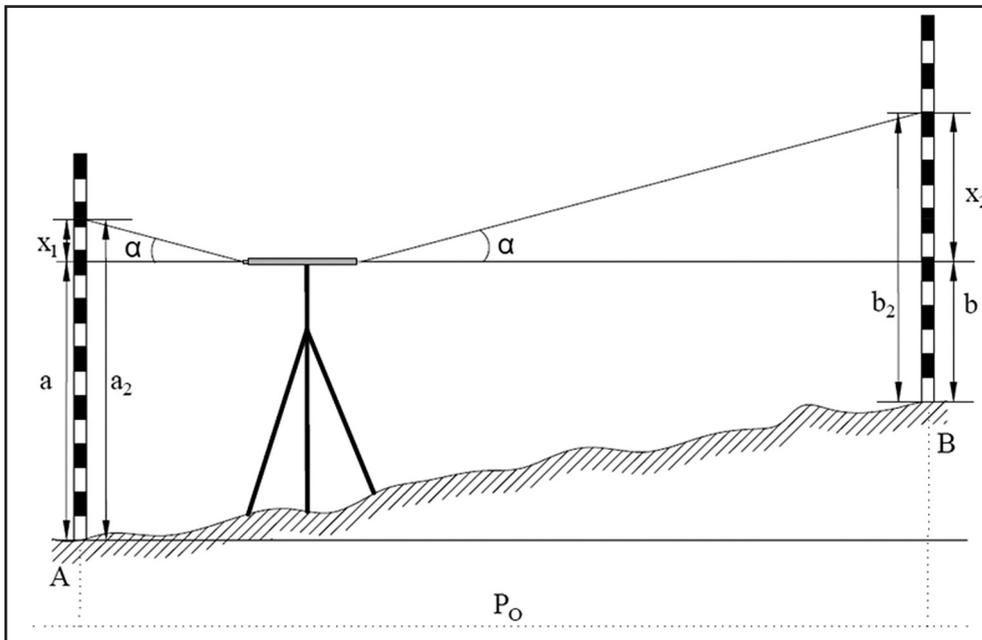The quantity is the deviation from the true value of the correct height difference.

Fig. 2. Illustration of the reading error on the staff in the case of geometric levelling where the instrument is not positioned at the centre (Păunescu *et al.*, 2019).

2.2. Calculation of the residual collimation error values (the non-parallelism between the sighting axis and the axis of the circular level) depending on the instrument's position between the two staffs

As specified in the previous section, the instrument must be placed at the centre of the levelling, i.e. so that the two spans are equal. The more unequal the spans (sections) are, the greater the collimation error. Table 1 presents the error values, depending on the inclination of the sighting axis and the length of the sections. From figure 3, we can observe the right-angled triangle ABC, in which:
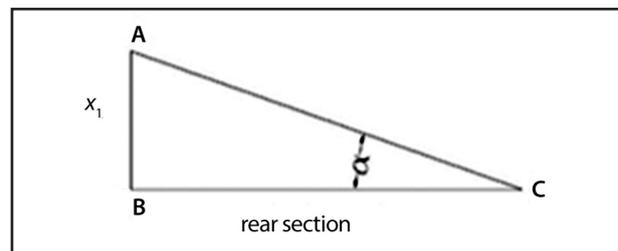

Fig. 3. Calculation of the error of non-parallelism.

- point A represents the erroneous reading on the staff, due to the non-parallelism;
- point B represents the correct reading (if angle did not exist);

- point C represents the position of the instrument.

  Thus:
- angle is the tilting angle of the sighting axis;
- distance CB is the length of the span;
- distance AB is the error (or, depending on the staff on which the reading is made), calculated by the formula:

$$x_1 = rear\,section(distance\,BC) \times \tan \alpha \qquad (5)$$

In table 1, we have the values with which we have calculated the reading errors on the staff, specifically the values x_1 and x_2 from figure 3, using formula (5). Error x_1 is calculated for the back staff, and x_2 error is calculated for the fore staff. We considered a tilting angle of the sighting axis of 10″ and a total levelling distance of 100 m.

The length of the levelling section is for a first-order network. For other orders, the levelling distance is increased. If we had kept the spans equal (i.e. 50 m each), we would not have had any collimation error, according to formula (3).

In order to calculate the magnitude of the reading errors on the staff as transmitted to the height difference, we simulated differences between the spans against the optimal value of 50 metres. The sum of the two spans will always be 100 metres, the total distance of the levelling station.

Following the formula (4), we conclude that difference between the true height difference and the incorrect one is given by $(x_2 - x_1)$, which is the difference between the two errors.

In table 1, the column called „Difference" is calculated in mm. This calculation is performed for a single station. However, geometric levelling involves many more stations, varying depending on the distance between the two end points and the gradient. For this reason, the error for a single station is multiplied by 10 for a levelling route with 10 stations, by 20 for 20 stations, and by 30 for a route with 30 stations.

Table 1. The values of the errors of non-parallelism

Size of the angle α	Section (m)	Back section (m)	Fore section (m)	Error x_1 on the rear section (mm)	Error x_2 on the front section (mm)	Difference $(x_2\text{-}x_1)$ (mm)	The cumulative difference on 10 stations (mm)	The cumulative difference on 20 stations (mm)	The cumulative difference on 30 stations (mm)
10″	100	49	51	0.0023	0.0025	0.097	0.97	1.939	2.9089
		47	53	0.0023	0.0026	0.2909	2.909	5.818	8.7266
		45	55	0.0022	0.0027	0.4848	4.848	9.696	14.544
		40	60	0.0019	0.0029	0.9696	9.696	19.39	29.089
		30	70	0.0015	0.0034	1.9393	19.39	38.79	58.178
		20	80	0.0010	0.0039	2.9089	29.09	58.18	87.266
		10	90	0.0005	0.0044	3.8785	38.79	77.57	116.36
		0	100	0	0.0048	4.8481	48.48	96.96	145.44

2.3. Results

With a span difference of only 1 metre from the ideal position (the centre), the error value in the height difference for a single station is only 0.097 mm, which is insignificant. Over 10 stations, the value is 0.97 mm; at 20 stations, it is 1.939 mm; and at 30 stations, it is 2.91 mm. According to the Technical Norms for the elaboration of the basic topographic plan at scales 1:2000, 1:5000 and 1:10000, approved by MAIA Order no. 147/12.12.1980, for first-order levelling, the span is 50 metres and the tolerance is $\pm 0.5\sqrt{L}$, where L is the route length in km.

In other words, there is a 1 mm misclosure allowed per km of route. It should be noted that, in addition to this, the other previously mentioned errors may also occur; therefore, over a large number of stations, the results may no longer fall within the admitted tolerance. Furthermore, if a difference

of only 1 m from the optimal span makes it difficult to meet tolerances, how would we manage if the difference reached 3 m, where even at 10 stations we would barely fall within the permitted limits?

In figure 4 is illustrates the case of the error that exceeds the tolerance. The solution is to send the levelling instruments for metrological calibration, to check them and determine if they are within parameters or require adjustment. The procedure for rectifying the measurement errors of these instruments is explained below. In the above-mentioned figure, the horizontal axis represents the number of stations and the vertical axis represents the error in mm. At the value of 1 mm, we have a line parallel to the horizontal axis, which represents the maximum allowable error – specifically the tolerance for first-order levelling of $\pm 0.5\sqrt{L}$. For 1 km of levelling, a misclosure of 1 mm is permitted between two measurements.
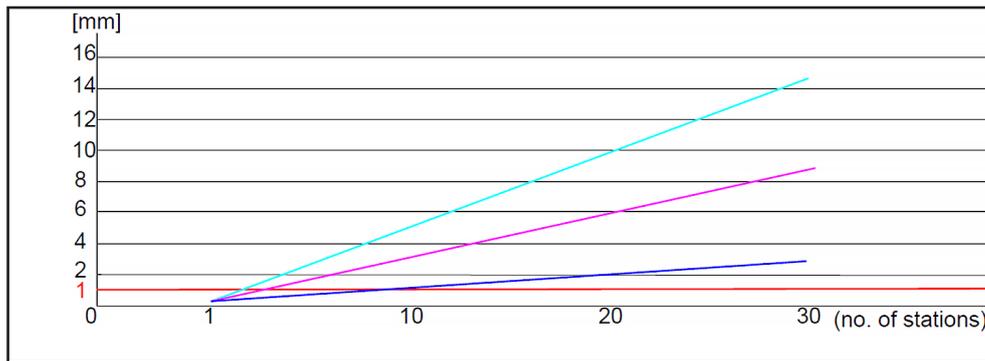
Fig. 4. Mechanism of collimation errors.

The blue line represents, in figure 4, the error calculated for a difference of 1 m between the two spans. It can be seen that, at 10 stations, the line intersects the tolerance; therefore, any value above this exceeds the tolerance. The pink line represents the error calculated for a difference of 3 m between the spans. We observe that at approximately 2 stations it intersects the tolerance, meaning any value above it exceeds the limit. Finally, the blue line represents the error calculated for a difference of 5 m between the two spans. To note that immediately after the first station, it intersects the tolerance, and any subsequent value exceeds it.

3. RESIDUAL COLLIMATION ERROR

The residual error caused by the non-parallelism between the sighting axis and the axis of the circular level is considered one of the most critical instrumental errors in precision geometric levelling. This error, commonly referred to as collimation error, produces systematic deviations in height differences, particularly when the sight lengths to the back and fore staffs are unequal (Wolf and Ghilani, 2018).

When the instrument is positioned exactly at the midpoint between the two staffs, the effect of this error is largely compensated. Under ideal conditions, the errors affecting the back and fore readings are equal and cancel out when computing the height difference. However, when the instrument is displaced from the midpoint, the reading errors differ in magnitude, leading to an incorrect height difference.

Based on classical surveying theory, the magnitude of the reading error depends on the inclination angle of the sighting axis and the length of the sight. Analytical relations were used to compute the errors associated with different configurations of back and fore sight lengths, considering a constant inclination angle of the sighting axis. The numerical simulations demonstrate that even small deviations from equal sight lengths can lead to cumulative errors that exceed admissible tolerances when a large number of stations is involved (Torge and Müller, 2012). The results clearly show that the inequality of sight lengths represents a major risk factor in precision levelling, especially in first-order networks, where strict tolerances are imposed.

3.1. ADJUSTMENT OF LEVELLING INSTRUMENTS: GEOMETRIC CONDITIONS OF THE STRUCTURAL AXES

In order for these geometric levelling instruments to be used under optimum measurement conditions, they must meet the following minimum requirements:
*   the main vertical axis „VV" must always be vertical, satisfying the condition: VV || L'L';
*   the sighting axis of the telescope „OO" must be parallel to the axis of the circular level „LL" (the collimation condition);
*   the sighting axis and the axis of the circular level must lie in the same vertical plane (to avoid crossover error);
*   it is advisable for the reticule (cross-hairs) to be correctly positioned within the eyepiece mount;
*   the focusing system must be perfectly functional to ensure the height of the sighting axis remains constant (Dragomir, 2009; Onose *et al.*, 2014; Berg and Holliday, 2011; INM 2020a and 2020b).
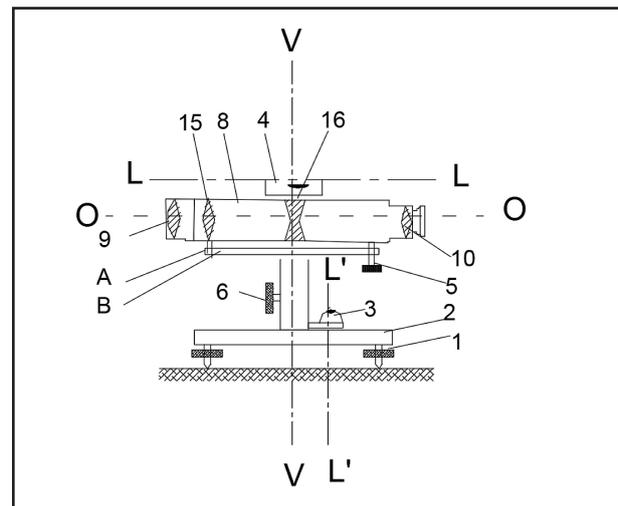

Fig. 5. Structural diagram of a level (Păunescu *et al.*, 2019).

3.2. ABSOLUTE VERIFICATION METHOD

The determination of the standard deviation per kilometre of double levelling can be performed using the method of geometric levelling from the centre, by sighting

two staffs positioned at a distance from the instrument. Furthermore, this distance and the type of staffs used are established according to the precision class of the instrument being tested, as specified in table 2 (Duţă, 2008; INM 2020a and 2020b).

Table 2. Selection of staff type according to the precision class of the instrument under test

Precision class	Length of the sections [m]	Type of target	Inequalities allowed between sections [m]
1	15	invar	± 0,25
2	15	invar	± 0,5
3	25	wood	± 1,0
4	25	wood	± 2,0

It is not recommended for deviations in the horizontal plane, relative to the station point, to exceed 0.25 m from the line joining the end points. It is recommended that the inequality of the spans does not exceed the values specified in table 2 and, regarding the maximum gradient between points, it should not exceed 30%.

It is recommended to perform at least 10 series of measurements, each consisting of 10 height difference determinations. The transition from one series to another is performed by changing the height of the instrument (the station must be reset). The final operation involves calculating the standard deviation using the following relation:

$$m_{st} = \pm \sqrt{\left(\frac{m_1 + m_2 + \dots + m_j + \dots m_g}{g}\right)^2} \qquad (6)$$

where:

g – represents the number of measurement series (10 series);

m_j – represents the standard deviation (root mean square error) of a height difference measurement within the j-th station of the series.

The error value is calculated using the following relation:

$$m_j = \pm \sum \frac{(\Delta h)}{10} \qquad (7)$$

where:

Δh – represents the residual (the deviation of the height difference from the arithmetic mean of the respective series)).

The standard deviation per kilometre of double levelling, denoted as m_{km}, is calculated using the following relation:

$$m_{km} = \pm m_{st} \sqrt{\frac{n}{2}} \qquad (8)$$

where:

n – represents the number of stations per kilometre of double levelling, which is calculated with the formula:

$$n = \frac{1000}{2d} \qquad (9)$$

d - is the distance in metres from the instrument to the staff (the span length). The m_{km} value determined in this manner must not, under any circumstances, exceed the specified

tolerance (Duţă, 2008; Featherstone *et al.*, 1998; INM, 2020a and 2020b).

4. ANALYSIS OF ERROR ACCUMULATION

The cumulative effect of the residual collimation error becomes significant when levelling routes include a large number of stations. Although the error introduced at a single station may appear negligible, its accumulation over multiple stations can result in misclosures that exceed the allowable limits specified by technical standards.

Numerical simulations performed for different configurations of span inequality illustrate the rapid growth of cumulative error as the number of stations increases. These results confirm that maintaining approximately equal back and fore spans is essential for ensuring compliance with tolerance requirements in high-precision levelling.

According to classical technical norms for first-order levelling, the admissible misclosure is proportional to the square root of the route length. When additional instrumental and observational errors are considered, the importance of minimising the residual collimation error becomes even more evident.

5. METROLOGICAL VERIFICATION AND INSTRUMENT RECTIFICATION

To ensure reliable results in precision geometric levelling, instruments must satisfy strict geometric and functional conditions. These include the verticality of the main vertical axis, the parallelism between the sighting axis and the circular level axis, and the correct alignment of the reticule. Any deviation from these conditions leads to systematic errors that cannot be fully detected through field redundancy alone (Duţă, 2008).

Metrological verification procedures, carried out in specialised laboratories, allow for the detection and adjustment of such instrumental defects. These procedures involve both absolute and relative testing methods and are adapted to the precision class of the instrument under examination. International and national recommendations require that precision levelling instruments undergo periodic verification, typically every two years or whenever abnormal behaviour is observed in the field (INM Specific Procedure; IAG Guidelines). The issuance of a calibration certificate following verification provides essential information on instrument.

6. CONCLUSIONS

This paper emphasises the essential role of precision geometric levelling in the establishment of gravimetric networks. Although gravimetric measurements are performed using specialised instruments, their accuracy is heavily dependent on the quality of the vertical reference provided by levelling.

The „3 µGal rule" is well known: an altitude determination error of 0.01 metres results in a gravity error of approximately 3 µGal. Since certain mineral exploration tasks require an accuracy of 10-20 µGal, an altitude error of 0.03-0.07 metres can lead to erroneous results due to the noise induced by height uncertainty.

The analysis demonstrates that the residual error caused by the non-parallelism of the sighting axis can accumulate rapidly when span lengths are unequal, leading to significant deviations from admissible tolerances. The results highlight the necessity for strict field procedures, proper instrument handling, and regular metrological verification. By integrating classical surveying theory with analytical simulations, this study underlines the importance of metrology in maintaining the accuracy and reliability of gravimetric networks, supporting the continued use of precision geometric levelling in high-accuracy geodetic and geophysical applications.

This case study highlights the role of geometric levelling in the development of gravimetric networks. Theoretical aspects are complemented by practical examples regarding the errors in geometric levelling that can influence the accuracy of gravimetric networks nationally. Furthermore, it emphasises the importance of using measuring instruments that undergo periodic verification, highlighting the critical role of the metrological verification process.

As shown in figure 4, errors can occur due to a failure to maintain equal spans, depending on the magnitude of the inequality. It is vital for instruments to be checked to ensure there are no non-parallelism errors between the sighting axis and the axis of the circular level. The primary objective of this study is to emphasize that the determination of height differences must be conducted as rigorously as possible, accounting for all potential measurement errors. Metrology assists users of these devices through adjustment and the minimisation of measurement errors.

If the instrument passes the verification, a calibration certificate is issued, specifying the deviations of angular measurements in the horizontal plane and the standard deviation of the height difference per kilometre of double levelling. These structural errors can be corrected using specific adjustment methods in the specialised laboratories of the National Institute of Metrology, or in laboratories accredited to perform periodic metrological verifications by authorised personnel. It is necessary for these instruments to be verified periodically, i.e., every two years, or more frequently whenever required by the user. By definition, metrology detects and adjusts for measurement errors, ensuring the instrument is used correctly and complies with the optimum tolerances specified by the manufacturer.

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